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AN APPROACH TO THE THEORY OF ERRORS IN SPACE NAVIGATION

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FOREWORD

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AN APPROACH TO THE THEORY OF ERRORS IN SPACE NAVIGATION

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Statement of the Problem. Astronavigation can serve as one of the principal means for the autonomous determination of the coordinates of the position of a flying apparatus in flight in cosmic space. Individual problems of the application of astronomical measurements in cosmic navigation are examined in cited works 1-3. It was shown that the use of automatic phototracking systems for tracking celestial bodies would permit sufficiently accurate determination of positions by means of astronomical measurements made on board a flying apparatus. The accuracy of tracking achieved, which was on the order of 0.1"-0.3" /See Note/ makes it possible to determine the coordinates of the position with a relative error of 1:10,000 which is fully adequate for interplanetary flights. However, the speed of a flying apparatus calculated by astronomical measurements was obtained with inadmissible errors. Therefore, in order to determine all navigation elements, the astronavigational system will have to be supplemented by, for example, an inertial system /4/ for determining the speed of flight. (Note: Astronomic orientator -- an astronomic navigational system providing the measurement of the coordinates of the position of a flying apparatus.)

The errors in an astronavigational system are determined not only by errors in direction finding, but also errors in the ephemerides of celestial bodies, errors in measuring time, and the geometry of the method for measuring elements of navigation.

A generalization of possible methods for astronomical interplanetary navigation is given in this paper and an evaluation of errors in these methods is presented here.

We shall call all the bodies within the bounds of the solar system (the sun, planets) celestial bodies.

1. The Principles of Interplanetary Astronavigation. The use of surfaces of position, the geometrical places of points of the probable position of a flying apparatus, constitutes the basis of astronomical methods of interplanetary navigation. Three surfaces of

position are required for determining the coordinates of a flying apparatus. The intersection of two surfaces gives a line of position and the intersection of this line with a third surface gives a series of points, one of which corresponds to the position of the flying apparatus. The actual point of the position can be distinguished from the fictional ones on the basis of knowledge of the approximate position.

Different geometrical relationships are used as surfaces of position and these relationships can be obtained by means of astronomical measurements on board a flying apparatus. With the aid of a phototracking system, an astronomical orientator makes it possible to measure the following angular astronomical parameters: the direction to the centers of celestial bodies, the angles between the directions to these centers and the angular measurements of the diameters of celestial bodies. On this basis it is possible to construct surfaces of equal diameters of celestial bodies, equal angles between a celestial body and a star, and equal angles between celestial bodies.

A surface of equal diameters is obtained by measuring the angular dimension β (Figure 1a) of the apparent diameter of a celestial body. The connection between the measured angle β, the known diameter D, and the distance H from the celestial body to the flying apparatus is given by the equation

$$\left(\frac{D}{2 \sin \beta/2}\right)^2 = R^2 = (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 \qquad (1.1)$$

where x, y, z, x_1, y_1, and z_1 are the coordinates of the flying apparatus and the celestial body respectively. If β = const., then the expression (1.1) is the equation of a spherical surface.

A surface of equal angles between a celestial body and a star is obtained by measuring the angle α between the direction to a star and the center of the celestial body (Figure 1b). Inasmuch as the rays from a star are practically parallel, the surface of position has the form of a cone with its vertex in the center of the celestial body. If φ, t_M and δ, t_s are the latitude and longitude of the flying apparatus and the star respectively in a system of polar coordinates with the origin in the center of the planet, then the equation of the surface of equal angles α will be

$$\cos \alpha = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos (t_s - t_M) \qquad (1.2)$$

The third surface of position is obtained as a result of measuring the angle α between the centers of two celestial bodies (Figure 1c). The surface is a toroid obtained by revolving the arc of a circle about the H_1 - H_2 axis connecting the centers of the planets. The equation of the toroid has the form

$$l^2 = R_1^2 + R_2^2 - 2R_1R_2 \cos \alpha \tag{1.3}$$

where R_1 and R_2 are the distances from the planet to the flying apparatus and l is the distance between the planets.

The three types of surfaces of position examined here can be used to set up nine methods for astronomical navigation in cosmic space. Some of these methods will be examined in more detail.

2. Systematic Errors in Measuring Navigational Elements. It follows from equations (1.1)-(1.3) that in order to obtain the surfaces of position it is essential to know the dimensions of the celestial bodies, their ephemerides, and the angular coördinates of stars in addition to measuring the angles between celestial bodies and stars and the angular diameters of celestial bodies. At the same time, both the measured quantities as well as the dimensions of celestial bodies, their ephemerides, the distances between the bodies, and the angular coordinates of stars introduced in equations (1.1)-(1.3) are known, with errors. We shall examine these errors in more detail.

The absolute distances between celestial bodies is known with an error on the order of 10^{-4} of the measured quantity. As a result, the coordinates of the position of the planets and their angular coordinates have not been determined precisely. When measuring the direction to the center of the planet, the phototracking system is aimed at the center of its brightness, the position of which shifts, depending on the phase of the planet. This error grows as the flying apparatus approaches the celestial body on which it is bearing.

Taking angular measurements of the apparent diameter of a celestial body is connected with errors caused by different illumination of its surface, the effect of the atmosphere of the celestial body, departure from a spherical shape, and the corona (for the sun). For example, if the earth were used as a navigational body, the error in measuring its diameter might reach 1/300.

In direction finding with stars, the errors would occur as a consequence of aberrations in the light and the proper movement of the stars. The values of the parallax and the proper movement of different navigational stars are given in the table.

The angular displacement of a ray θ caused by aberration of light is determined by the formula

$$\operatorname{tg} \theta = \frac{V}{c} \sin \psi \tag{2.1}$$

where ψ is the angle between the direction of the vector of the velocity of flight V and the direction to the star, c is the velocity of light. With a flight velocity on the order of 100 kilometers per second, the angle θ can reach values of 1'.

Selection of double stars as navigational stars will give rise to error from variation in the brightness of the stars composing the double star.

Star	Brightness (magnitude)	Parallax (micro-radians)	Proper Motion (microradians per year)
Sirius	-1.58	1.9	6.6
Canopus	-0.86	0.07	0.11
α Centauri	0.06	3.8	18.4
Vega	0.14	0.6	1.7
Capella	0.21	0.35	2.2
Arcturus	0.24	0.4	11.4
Rigel	0.34	0.03	0.03
Procyon	0.48	1.6	1.6
Achernar	0.60	0.25	0.5
β Centauri	0.86	0.08	0.2
Altair	0.89	1.0	3.3
Betelgeuse	0.92	0.08	0.16
α Crucis	1.05	0.07	0.24
Aldebaran	1.06	0.3	1.0
Pollux	1.21	0.5	3.1
Spica	1.21	0.07	0.3
Antares	1.22	0.05	0.16
Fomalhaut	1.29	0.7	1.8
Deneb	1.33	0.03	0.02
Regulus (α Leo)	1.34	0.3	1.22
β Crucis	1.50	0.08	0.27

The finite speed of light is also a cause for errors in navigation since the planet on which a bearing is being taken will be displaced in its orbit during the time the light is traveling from the planet to the observer. Thus, for example, if a flying apparatus is 143 million kilometers from Mars, the light will cover this distance in 480 seconds, but Mars will have moved more than 11,000 kilometers in this time.

3. <u>A Method of Navigation Based on Measuring the Diameters of Three Celestial Bodies</u>. By taking angular measurements β_1, β_2, β_3 of three celestial bodies H_1, H_2, H_3 with diameters D_1, D_2, D_3 it is possible to construct three spherical surfaces of position. Let x_i, y_i, and z_i be the coordinates of the centers of celestial bodies and x, y, and z be the coordinates of a flying apparatus M (Figure 2). From the geometrical construction of three spherical surfaces of position one may determine two points M and M_1 (Figure 3), one of which corresponds to the actual position of the flying apparatus and the second is a fictitious position. With a great distance between the points M and M_1 it is possible to distinguish the actual point from the fictitious one by the approximate value of the position.

The selection of the celestial bodies should be such that the points M and M_1 will not turn out to be too close to each other during the flight.

The coordinates x, y, and z of the flying apparatus can be determined analytically by solving the following equations simultaneously

$$\frac{D_1^2}{4\sin^2\frac{\beta_1}{2}} = (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = R_1^2$$

$$\frac{D_2^2}{4\sin^2\frac{\beta_2}{2}} = (x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2 = R_2^2 \qquad (3.1)$$

$$\frac{D_3^2}{4\sin^2\frac{\beta_3}{2}} = (x - x_3)^2 + (y - y_3)^2 + (z - z_3)^2 = R_3^2$$

A computer should be on board to solve these equations so that measured angular data of the celestial bodies, their diameters and their ephemerides, can be fed into it.

Let us determine the errors Δx, Δy, Δz of the astronomical system, assuming that the quantities D_i, β_i, x_i, y_i, z_i $(i = 1, 2, 3)$ are introduced with errors ΔD_i, $\Delta\beta_i$, Δx_i, Δy_i, Δz_i. From equations (3.1) we obtain

$$\frac{\Delta R_i}{R_i} = \frac{\Delta D_i}{D_i} - \frac{\Delta\beta_i}{2}\,\mathrm{ctg}\,\frac{\beta_i}{2} \qquad (i = 1, 2, 3) \qquad (3.2)$$

where

$$\frac{\Delta R_i}{R_i} = (\frac{x - x_i}{R_i})^2 \frac{\Delta x - \Delta x_i}{x - x_i} + (\frac{y - y_i}{R_i})^2 \frac{\Delta y - \Delta y_i}{y - y_i} + (\frac{z - z_i}{R_i})^2 \frac{\Delta z - \Delta z_i}{z - z_i} \tag{3.3}$$

For small angles β we have $\beta \approx D/R$, $\sin\beta \approx \beta$, $\cos\beta \approx 1$, therefore

$$\Delta R_i = -\frac{R_i^2}{D} \Delta\beta \tag{3.4}$$

It follows from this that the error ΔR_i grows proportionally with the square of the distance to the celestial body and decreases when one is finding the direction of celestial bodies of large diameters.

It is obvious that there exist limit distances to the celestial bodies whose directions we wish to find and that these limits are set by the resolving capacity of the phototracking system. If $\Delta\beta_0$ is the angle within whose limits any changes in the apparent diameter D cannot be detected by the phototracking system, then the limit distance from the celestial body will be

$$R_{limit} \leq \frac{D}{\Delta\beta_0} \tag{3.5}$$

Thus, for example, when $\Delta\beta_0 = 10^{-3}$ radians, the limit distance from the earth as a navigational celestial body is 12.7 million kilometers.

The connection between the errors $\Delta x, \Delta y, \Delta z$ and the initial errors can be represented in the form of the matrix equation

$$\Delta r [\bar{r} - \bar{r}] = \Delta\bar{r}[\bar{r} - \bar{r}] + \bar{R}\Delta\bar{R} \tag{3.6}$$

where

$$\Delta r = \begin{matrix} \Delta x \\ \Delta y \\ \Delta x \end{matrix} \qquad \Delta\bar{r} = \begin{matrix} \Delta x_1 \\ \Delta y_1 \\ \Delta z_1 \end{matrix} \qquad [\bar{r} - \bar{r}] = \begin{matrix} x - x_1 & y - y_1 & z - z_1 \\ x - x_2 & y - y_2 & z - z_2 \\ x - x_3 & y - y_3 & z - z_3 \end{matrix} \tag{3.7}$$

$$\bar{R}\Delta\bar{R} = \begin{matrix} R_1 \Delta R_1 \\ R_2 \Delta R_2 \\ R_3 \Delta R_3 \end{matrix}$$

Let us examine the effect of the mutual arrangement of the celestial bodies on navigation errors. For this purpose we shall

take a coordinate system with its origin at the center of mass of the flying apparatus and the z-axis will point toward the center of the celestial body Π_1 (Figure 4). For the case of flight in the direction of the z-axis we obtain $x = y = z = 0$, $x_1 = y_1 = 0$, $z_1 = R_1$

$$x_2 = R_2 \sin\alpha_1 \cos u_2 \qquad y_2 = R_2 \sin\alpha_1 \sin u_2 \qquad z_2 = R_2 \cos\alpha_1$$
$$x_3 = R_3 \sin\alpha_3 \cos u_3 \qquad y_3 = R_3 \sin\alpha_3 \sin u_3 \qquad z_3 = R_3 \cos\alpha_3$$

$$\Delta x = - \frac{(R_1 \Delta R_1 + \Delta F_1)(\sin\alpha_1 \cos\alpha_3 \sin u_2 - \cos\alpha_1 \sin\alpha_3 \sin u_3)}{R_1 \sin\alpha_1 \sin\alpha_3 \sin (u_3 - u_2)} - \frac{(R_2 \Delta R_2 + \Delta F_2) \sin u_3}{R_2 \sin\alpha_1 \sin (u_3 - u_2)} + \frac{(R_3 \Delta R_3 + \Delta F_3) \sin u_2}{R_3 \sin\alpha_3 \sin (u_3 - u_2)} \tag{3.8}$$

$$\Delta y = \frac{(R_1 \Delta R_1 + \Delta F_1)(\sin\alpha_1 \cos\alpha_3 \cos u_2 - \cos\alpha_1 \sin\alpha_3 \cos u_3)}{R_1 \sin\alpha_1 \sin\alpha_3 (u_3 - u_2)} + \frac{(R_2 \Delta R_2 + \Delta F_2) \cos u_3}{R_2 \sin\alpha_1 \sin (u_3 - u_2)} - \frac{(R_3 \Delta R_3 + \Delta F_3) \cos u_2}{R_3 \sin\alpha_3 \sin (u_3 - u_2)}$$

$$\Delta z = \Delta R_1 + \frac{\Delta F_1}{R_1}$$

It follows from the expressions of (3.8) that as $\alpha_1 \to 0$, and $\alpha_3 \to 0$ the errors increase without bound. An analogous situation prevails when $u_3 - u_2 \to 0$. Consequently, angles α_1 and α_2 between the directions to the centers of the celestial bodies should not be less than definite quantities. The smallest errors are obtained when α_1, $\alpha_3 \approx 90$ degrees and $u_3 - u_2 = 90$ degrees.

4. A Method of Navigation Based on Measuring the Diameters of Two Celestial Bodies and the Direction to a Star. This method consists of measuring the angles β_1 and β_2 of the apparent diameters of celestial bodies and the angle α between the direction to the star and the center of one of the celestial bodies (Figure 5). The possible position of the flying apparatus is obtained in the form of two points of intersection of two spheres with radii R_1 and R_2 and a cone with an angle of 2α at the apex.

It is possible to make use of a coordinate system x', y', and z' with its origin at the center of one of the celestial bodies and one of its axes pointed to the reference star for the analytical determination of the coordinates of a flying apparatus. In this coordinate system the equations of the surfaces of position will be

$$\frac{D_1^2}{4\sin^2\frac{\beta_1}{2}} = (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = R_1^2$$

$$\frac{D_2^2}{4\sin^2\frac{\beta_2}{2}} = x^2 + y^2 + z^2 = R_2^2 \qquad x^2 + y^2 = z^2\,\mathrm{tg}^2\,\alpha \tag{4.1}$$

where x_1, y_1, z_1 are the coordinates of the center of the celestial body Π_1.

Equations (4.1) can be used to obtain the dependence of the errors of navigation Δx, Δy, Δz on the initial errors in measuring the angles of the apparent diameters $\Delta\beta_1$ and $\Delta\beta_2$, the angle between the star and the celestial body $\Delta\alpha$, the diameters ΔD_1 and ΔD_2 and the ephemerides Δx_1, Δy_1, Δz_1. We shall find

$$(x - x_1)\Delta x + (y - y_1)\Delta y + (z - z_1)\Delta z = a_1$$

$$x\Delta x + y\Delta y + z\Delta z = a_2 \qquad x\Delta x + y\Delta y - z\,\mathrm{tg}^2\alpha\,\Delta z = a_3 \tag{4.2}$$

where

$$a_1 = \left(\frac{\Delta D_1}{D_1} - \frac{\Delta\beta_1}{2}\,\mathrm{ctg}\,\frac{\beta_1}{2}\right) R_1^2 + (x - x_1)\Delta x_1 + (y - y_1)\Delta y_1 + (z - z_1)\Delta z_1 \tag{4.3}$$

$$a_2 = \left(\frac{\Delta D_2}{D_2} - \frac{\Delta\beta_2}{2}\,\mathrm{ctg}\,\frac{\beta_2}{2}\right) R_2^2 \qquad a_1 = z^2\,\frac{\mathrm{tg}\,\alpha}{\cos^2\alpha}\,\Delta\alpha$$

Solving the equations (4.2), we obtain

$$\Delta x = \frac{y\left[a_2 - a_1 + \frac{z_1}{z}(a_3 - a_2)\cos^2\alpha\right] - y_1(a_2\sin^2\alpha + a_3\cos^2\alpha)}{x_1 y - y_1 x}$$

$$\Delta y = \frac{x\left[a_1 - a_2 + \frac{z_1}{z}(a_2 - a_3)\cos^2\alpha\right] + x_1(a_2\sin^2\alpha + a_3\cos^2\alpha)}{x_1 y - y_1 x} \tag{4.4}$$

$$\Delta z = \frac{a_2 - a_3}{z}\cos^2\alpha$$

In order to evaluate the effect of the mutual arrangement of the celestial bodies and the star on the errors of navigation, we shall transform the expressions (4.4) by introducing spherical coordinates (Figure 6). After completing the necessary transformations we shall obtain

$$\Delta x = \frac{F_1}{\sin\alpha \sin\alpha_1 \sin (u - u_1)} \quad \Delta y = \frac{F_2}{\sin\alpha \sin\alpha_1 \sin (u - u_1)} \tag{4.5}$$

$$\Delta z = \left(\frac{\Delta D_2}{D_2} - \frac{\Delta\beta_2}{2} \operatorname{ctg} \frac{\beta_2}{2} - \Delta\alpha \sin\alpha\right) R_2 \cos\alpha$$

where F_1 and F_2 are functions which remain finite when $\alpha_1 \to 0$, $\alpha \to 0$, $u - u_1 \to 0$.

It follows from the expressions (4.5) that if the angles between the directions to the celestial bodies $\alpha_1 \to 0$, $\alpha \to 0$, and $u - u_1 \to 0$, then the errors Δx and Δy grow without bound. The selection of navigational celestial bodies and star should meet the requirement that the angles α, α_1, and $u - u_1$ should not be less than 20-30 degrees. The second restriction on the method examined here is the distance to the celestial bodies, which should not exceed the limit values.

5. A Method of Navigation Based on Measuring the Diameter of a Celestial Body and the Directions to Two Stars. The angles α_1 and α_2 between the directions to the stars S_1 and S_2 and to the center of the celestial body Π, also the angle β of the apparent diameter of this body (Figure 7) are measured in flight. As a result of the measurements we obtain two conical surfaces of position with a common vertex in the center of the celestial body Π and a spherical surface of position of radius R. The lines of intersection of the cones with the sphere are circles at whose intersections the two points M and M' are obtained.

One of these points is the actual position while the second is a fictitious one. This method is identical in its geometrical construction to the well-known method of terrestrial astronavigation based on measuring the zenith distances of two stars.

We shall set up a coordinate system at the center of the celestial body in order to determine the position analytically (Figure 8). In accordance with the notation of Figure 8, we obtain the equations of the surfaces of position from the polar triangles ZS_2M and ZS_1M

$$\cos\alpha_1 = \sin v \sin v_1 + \cos v \cos v_1 \cos (u_1 - u)$$

$$\cos\alpha_2 = \sin v \sin v_2 + \cos v \cos v_2 \cos (u_2 - u) \tag{5.1}$$

$$R = \frac{D}{2 \sin \beta/2}$$

where R, u, v are the spherical coordinates of the flying apparatus while u_1, u_2, and v_1, v_2 are the spherical coordinates of the stars.

The required coordinates R, u, v of the flying apparatus are determined from the equations (5.1).

We shall find the errors of navigation ΔR, Δv, Δu by expressing them through the errors of direction finding $\Delta\alpha_1$, $\Delta\alpha_2$, $\Delta\beta$, the errors in the coordinates of the stars Δu_1, Δu_2, Δv_1, Δv_2 and the error in the diameter ΔD. By making use of the equations (5.1), we find

$$\Delta R = R\left(\frac{\Delta D}{D} - \frac{\Delta\beta}{2}\operatorname{ctg}\frac{\beta}{2}\right)$$

$$\Delta v = \frac{(\Delta\alpha_2 + \Delta v_2 \cos q_2)\sin A_1 - (\Delta\alpha_1 + \Delta v_1 \cos q_1)\sin A_2}{\sin(A_2 - A_1)} \qquad (5.2)$$

$$\Delta u = \frac{(\Delta\alpha_1 + \Delta v_1 \cos q_1)\cos A_2 - (\Delta\alpha_2 + \Delta v_2 \cos q_2)\cos A_1}{\cos v \sin(A_2 - A_1)}$$

where A_1 and A_2 are azimuths and q_1 and q_2 are the parallactic angles of the stars determined from the equations

$$\cos v \sin v_i - \sin v \cos v_i \cos u_i = -\cos A_i \sin\alpha_i$$

$$(i = 1.2) \qquad (5.3)$$

$$\sin v \cos v_i - \cos v \sin v_i \cos u_i = \cos q_i \sin\alpha_i$$

It follows from the expressions of (5.2) that when $A_2 - A_1 \to 0$ the error grows without bound. Consequently, the difference in the azimuths of the stars should be different from zero. Ordinarily it is sufficient to have $A_2 - A_1 \geqslant$ 20-30 degrees.

At great distances from the celestial body the angle β is small, therefore, as may be seen from the first equation of (5.2), the error R becomes very large. For this reason it is expedient to use this method in flights near the celestial body.

6. A Method of Navigation Based on Measuring the Angles Between the Centers of Three Celestial Bodies. Measuring the angles α_1, α_2, α_3 between the directions to the centers of three celestial bodies (Figure 2) permits one to obtain three equations of surfaces of position

$$l_{12}^2 = R_1^2 + R_2^2 - 2R_1R_2\cos\alpha_1$$

$$l_{23}^2 = R_2^2 + R_3^2 - 2R_2R_3\cos\alpha_2 \qquad (6.1)$$

$$l_{31}^2 = R_3^2 + R_1^2 - 2R_1R_3\cos\alpha_3$$

where l_{12}, l_{23}, l_{31} are the distances between the centers of the celestial bodies.

The equations (6.1) are used to determine the distances R_1, R_2, R_3 which are connected with the coordinates of the flying apparatus by the equations

$$(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 = R_i^2 \qquad (i = 1, 2, 3) \qquad (6.2)$$

where x_i, y_i, z_i are the coordinates of the centers of the celestial bodies.

If $\Delta\alpha_1, \Delta\alpha_2, \Delta\alpha_3$ are the errors in measuring the angles and $\Delta l_{12}, \Delta l_{23}, \Delta l_{31}$ are the errors in the distances between the celestial bodies, then we obtain from (6.1)

$$(R_1 - R_2 \cos\alpha_1) \; R_1 + (R_2 - R_1 \cos\alpha_1)\,\Delta R_2 = l_{12}\,\Delta l_{12} - R_1R_2\,\Delta\alpha_1 \sin\alpha_1$$
$$(R_2 - R_3 \cos\alpha_2) \; R_2 + (R_3 - R_2 \cos\alpha_2)\,\Delta R_3 = l_{23}\,\Delta l_{23} - R_2R_3\,\Delta\alpha_2 \sin\alpha_2$$
$$(R_1 - R_3 \cos\alpha_3) \; R_1 + (R_3 - R_1 \cos\alpha_3)\,\Delta R_3 = l_{31}\,\Delta l_{31} - R_1R_3\,\Delta\alpha_3 \sin\alpha_3 \qquad (6.3)$$

From this we find

$$\Delta R_1 = \frac{\Delta_1}{\Delta} \qquad \Delta R_2 = \frac{\Delta_2}{\Delta} \qquad \Delta R_3 = \frac{\Delta_3}{\Delta} \qquad (6.4)$$

where

$$\Delta = \begin{vmatrix} R_1 - R_2 \cos\alpha_1 & R_2 - R_1 \cos\alpha_1 & 0 \\ 0 & R_2 - R_3 \cos\alpha_2 & R_3 - R_2 \cos\alpha_2 \\ R_1 - R_3 \cos\alpha_3 & 0 & R_3 - R_1 \cos\alpha_3 \end{vmatrix} \qquad (6.5)$$

while the determinants Δ_1, Δ_2, Δ_3 are obtained from (6.5) by replacing the corresponding columns with the right sides of the equations (6.3).

It follows from the determinants of (6.5) that when $\alpha_1 = \alpha_2 = \alpha_3 = 0$ $\Delta = 0$ and the errors of (6.4) grow without bound. The minimum values of the errors are obtained when $\alpha_1 = \alpha_2 = \alpha_3 = 90$ degrees.

7. A Comparative Evaluation of the Methods of Astronavigation. In addition, other methods based on other combinations of the previously cited surfaces of position or on the use of surfaces of position of a different geometrical form can be used in addition to the methods of astronavigation set forth here.

In all methods of navigation based on finding the directions of celestial bodies (planets), it is necessary to measure time and to know the ephemerides of the planets. Time can be measured by means of special clocks or by means of finding the direction of a fourth celestial

body with known ephemerides. The additional information obtained can also be utilized to decrease the errors of astronavigation and to eliminate the lack of uniqueness in the determination of position.

Each of these methods of navigation yields the best results under definite conditions. For long-range navigation it is expedient to make use of methods based on measuring the angles between the centers of three celestial bodies or two celestial bodies and a star. Short-range navigation, including landing a flying apparatus on the surface of a celestial body can be accomplished with the aid of a method based on measuring the angles between the center of the celestial body and two stars, and the apparent diameter of this body. Inasmuch as the selection of stars is very wide, the restrictions of the method in respect to the mutual arrangement of the stars and the celestial body are readily met.

When selecting a method of astronavigation, one should start with the requirements of minimum errors and simplicity of design of the astro-orientator. It should be taken into consideration that the design of a phototracking system which finds the directions of stars or celestial bodies in the form of illuminated points is less complicated than the design of a system which is to measure the apparent diameter of a celestial body. From this standpoint, those methods of navigation are preferable in which the angles between the centers of celestial bodies or stars are measured.

The methods of astronavigation do not permit direct measurement of flight velocities, which can be obtained only by differentiation of the coordinates. However, since the information on coordinates possesses high-frequency gaps, the flight velocity is obtained with inadmissible errors.

An astronavigational system should develop coordinates at the time of measurement, which is possible if the system is completely automatic.

In order to obtain an idea of the special features of the technical development, let us examine the structural diagram of a system of navigation which would realize the method based on measuring the apparent diameter of a celestial body and the directions to two stars (Figure 9). The solution of equations (5.1) would take place in this layout.

Telescopes T_1 and T_2 which are mounted on a gyroplatform and which are trained relative to the platform by means of a phototracking system find the directions of stars S_1 and S_2. The signals, which are proportional to the measured angles α_{1u} and α_{2u} are fed into a cosine mechanism. The obtained values of $\cos \alpha_{1u}$ and $\cos \alpha_{2u}$ are compared with the values of $\cos \alpha_{1v}$ and $\cos \alpha_{2v}$, which are computed in computers COM_1 and COM_2 on the basis of information fed into them. The mismatching signal obtained here is fed into amplifiers Y_1 and Y_2 and the adjusting motors D_v and D_u which produce the coordinates v and u.

Telescope T_3 measures the angular diameter of celestial body π and determines the direction to its center. The angular diameter is measured by having telescope T_3 scan in a cone, the angle of which is equal to the apparent diameter. The measured angle β_u is fed into a sine mechanism and computer COM_3 to compute R_u. Computer COM_4 computes R_4. The mismatching signal obtained from comparing signals R_u and R_v is fed into amplifier Y_3 and adjusting motor D_R which produces the coordinate R. The spherical coordinates R, v, and u can be recomputed into rectangular coordinates x, y, and z, for which an additional computer will be necessary.

The most important elements are shown in the diagram presented here (Figure 9). In fact, the diagram of the astro-orientator based on the use of equations (5.1) is more complicated. Some important connections are not shown in the diagram. In particular, it is not shown that the gyroplatform which is formed by a gyro-inertial system has to be corrected /4/ by feeding external information into the system. Such external information can be obtained from the astronomical navigational system. Consequently, the astronomical and gyro-inertial systems should be mutually connected. Under these circumstances, an astroinertial system will be formed which will possess the favorable properties of a gyro-inertial system (large memory and determination of velocities with sufficient accuracy) and those of an astronomical system (precise determination of coordinates).

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References

1. Larmore, L., "Celestial Observations for Space Navigation," Aero Space Engineering, January 1959

2. "Mathematical Devices in a Cosmic Rocket," Ekspress-informatsiya, Raketnaya tekhnika /Rocketry/, No. 23, 1959

3. "The Technology of Communications and Navigation in Interplanetary Flights," Voprosy radiolokatsionnoy tekhniki /Problems of Radar Technology/, No. 6, 1959

4. Bodner, V. A., and Seleznev, V. P., "An Approach to the Theory of Inertial Systems With Three Channels of Automatic Compensation for Accelerations from Gravitational Forces," Izv. AN SSSR, OTN, Energetika i avtomatika /Bulletin of the Academy of Sciences USSR, Department of Technical Sciences, Power Engineering and Automation/, No. 1, 1960

FIGURE APPENDIX

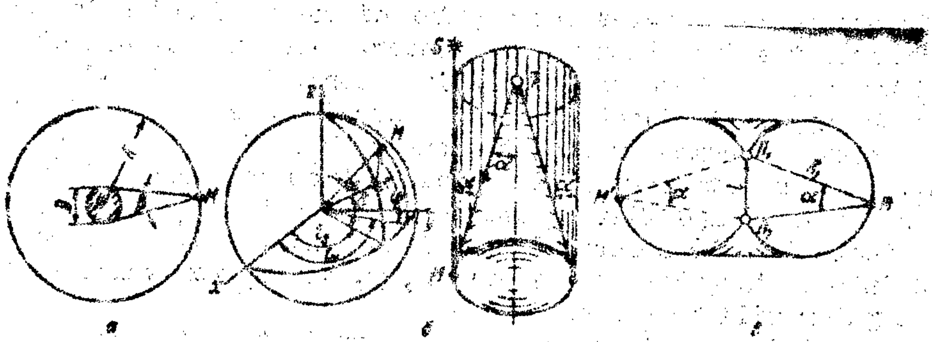


Figure 1.

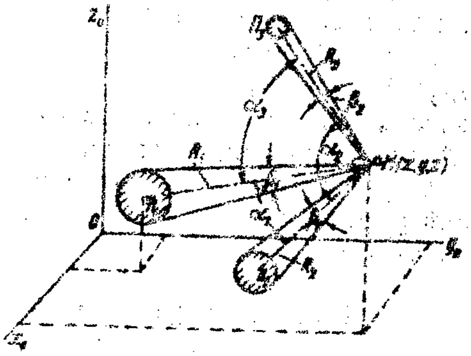


Figure 2.

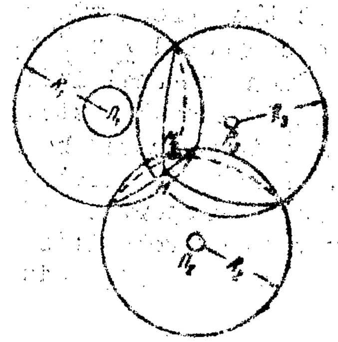
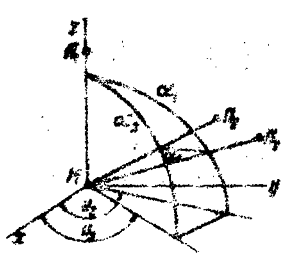


Figure 3.



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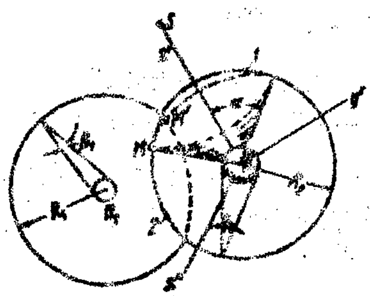


Figure 5.

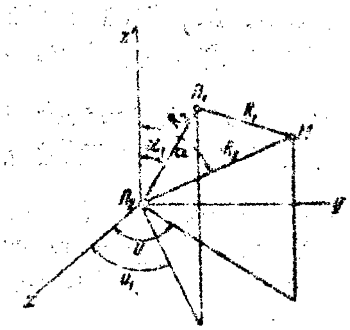
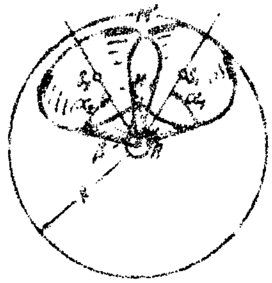


Figure 6.



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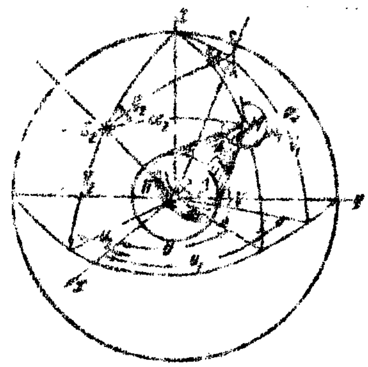

Figure 8.

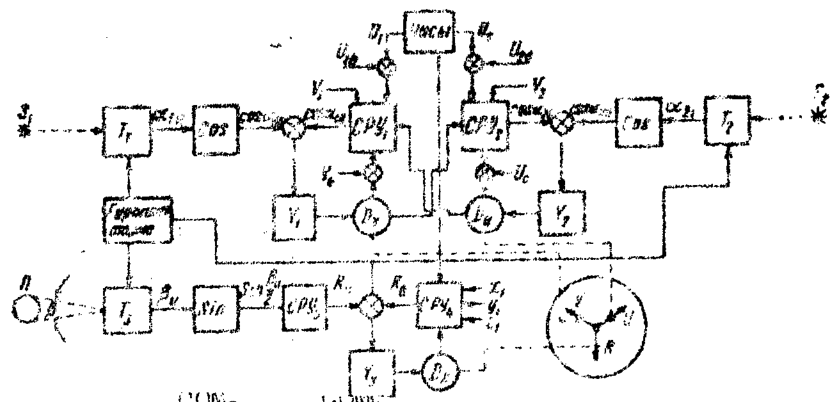

Figure 9.

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